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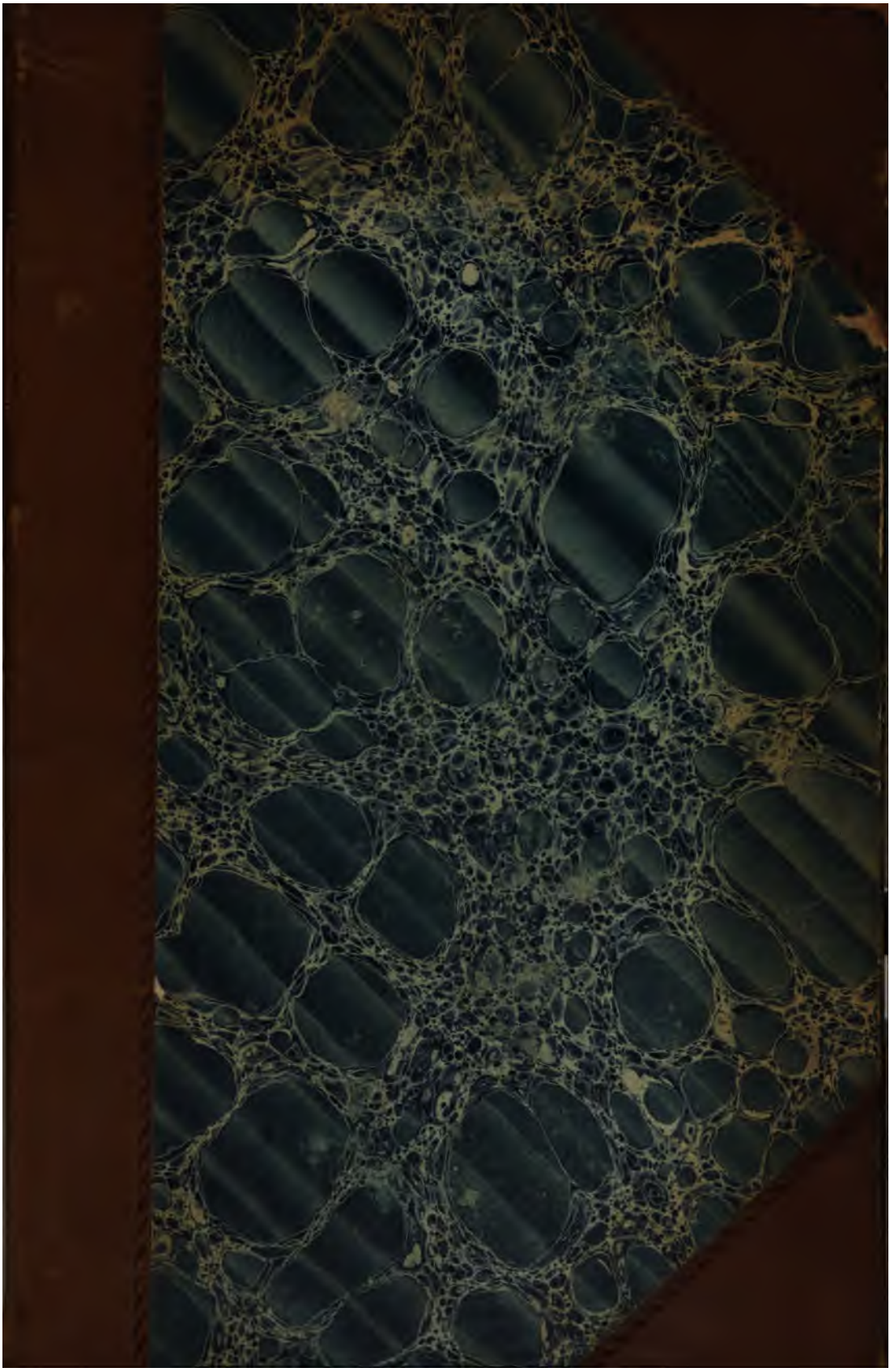

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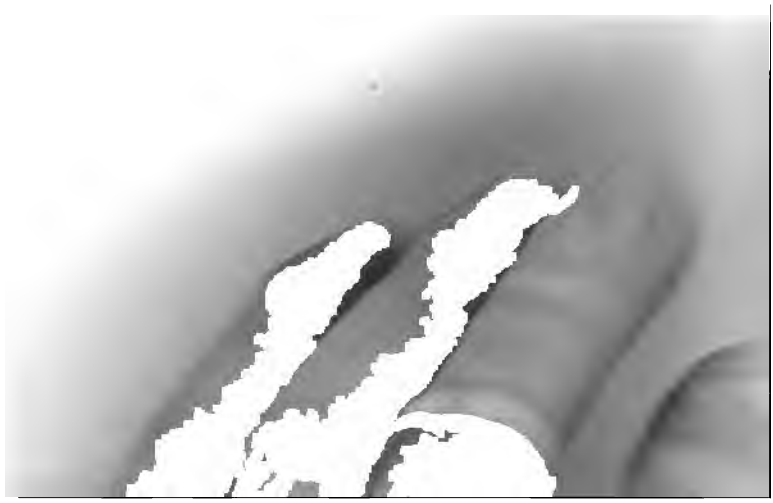

J. H. 1828

TABLES

SHOWING THE VALUES OF

ANNUITIES AND ASSURANCES

UPON

LIVES OF EQUAL AGES;

IN SINGLE AND ANNUAL PAYMENTS;

ACCORDING TO THE

Northampton Table of Mortality,

RATE OF INTEREST THREE PER CENT.

TO WHICH ARE ADDED,

EXAMPLES AND REMARKS ILLUSTRATIVE OF THE METHOD OF DETERMINING THE PRESENT VALUES OF THESE REVERSIONS ANALYTICALLY.

BY EDWARD HULLEY,

OF THE GLOBE ASSURANCE OFFICE.

London;

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TO THE

BOARD OF DIRECTORS

OF THE

GLOBE ASSURANCE OFFICE

This Work

IS MOST RESPECTFULLY DEDICATED,

BY THEIR

OBLIGED AND OBEDIENT

HUMBLE SERVANT,

E. HULLEY.

not be very readily understood by persons unacquainted with the doctrine of this subject.

It will appear then that, in the present instance, we have endeavoured to deliver, for the convenience of those who have made themselves acquainted with the nature and use of decimal fractions, and others, an immediate solution of those contingencies upon which Annuities and Assurances on lives, within the limits of Equal Ages, are usually granted; it will therefore be necessary to remind the reader that, upon finding the value of an Annuity or of an assigned Reversion from these Tables, which are computed at the rate of 3*l.* per cent. per annum compound interest, and for the whole duration of life, with regard to an Annuity, as contained in Tables I. to XXIII., the payment of 1*l.* is supposed to be made at the end of every year of its continuance; and with respect to an Assurance, as contained in Tables XXIV. to XLVIII., that the sum assured is payable at the end of the year in which the event entitling to it may happen or become fulfilled; and that the Annual Premiums are payable at the beginning of the year, and until the Annuity commences, or the Assu-

rance or reversionary sum becomes due, unless otherwise expressed, but in every case for the whole duration of the lives upon the continuance of which they may depend.

"Our knowledge of many things that interest "us the most amounts only to probability; our "persuasion of the uniformity of the course of "nature is only a high degree of it, and almost "all the judgments or decisions by which our "conduct is regulated in the affairs of life, are "solutions more or less accurate of problems in "the doctrine of probabilities."

"Generally, in such cases, we are satisfied "with a preponderance of probability on one "side, without attempting to determine its exact "measure, or the proportion it bears to certainty. "But where the receipt of a sum of money is "uncertain, it is frequently of great importance "to determine the exact measure of the proba-"bility that it will be received; since the value "of the sum dependent upon the contingency, is "less than if its receipt were certain, in the exact "ratio of that probability to certainty, and the "whole property of an individual is often dis-

"posed of according to estimates made upon "this principle." Tables of mortality founded upon actual observations for a series of years at different places have been constructed, which enable us to determine with a considerable degree of accuracy the probabilities of the happening of events affecting the future payment of given sums; a portion of the property of this country may be said to be contingent, or depending upon some given lives failing according to a prescribed order of precedency, that is, reverting only at the decease of some given life or lives, provided they survive, or become extinct during the existence of, some other given life or lives; and numerous Offices have been established, where, in consideration of the grant of immediate Annuities, the payment of Annuities or of given sums of money under such circumstances, may be secured; the values given in this work, it must be obvious, therefore, are, in our inquiries involving probabilities, among the most important of the numerical results, since they exhibit estimates regulated by the extent of the probabilities of the *happening* of human life, death, and survivorship, deducible only from

such Tables; whilst, in the other part of the work, the method of determining more particularly the probabilities just alluded to is considered.

In the execution of the present work accuracy and perspicuity have been principally studied; the examples herein given are the results of algebraical investigations, and are delivered free from analytical operations; amongst the Tables, which form by far the most comprehensive collection of the values, either in single or annual payments, of reversionary sums and Annuities involving three lives yet offered to the public, other forms of yearly payments are given than what would be ordinarily required, and of which no mention appears to have been yet made by writers upon these subjects, and one of which might oftentimes be found more convenient to adopt than another; and the values of Annuities for ever after the extinction of any given lives are introduced for the purpose of determining the values of perpetuities in reversion or reversionary freehold estates of given yearly values; and such Annuities might be exchanged by the several

Assurance Companies for the immediate Annuities or annual premiums stated in the Tables; upon the whole the author flatters himself that it will be found to contain information desirable in our present want of Tables upon these subjects.

We have been furnished by Mr. Morgan, in his "Doctrine of Annuities and Assurances," 1821, with the following Table, which gives a statement of the diseases of which persons assured by the Equitable Society, of which he is the Actuary, have died, during the course of 20 years, from 1800 to 1821, all certified by gentlemen of the medical profession; and which, although it may not have an immediate connection with the doctrine of this subject, may not improperly be here introduced; whether the proportion one disorder bears to another, or the proportion or share of Mortality befalling the several periods of human life from the total number of deaths, amongst the members of the Societies which have been established for granting Assurances on lives, may be determined therefrom, I leave to be considered.

NOSOLOGICAL TABLE.

DISEASE.	10 to 20.	20 to 30.	30 to 40.	40 to 50.	50 to 60.	60 to 70.	70 to 80.	80 &c.	TOTAL.
Angina Pectoris	—	—	5	11	12	9	4	3	44
Apoplexy	1	3	19	38	69	69	38	5	242
Asthma	—	—	—	2	19	19	11	2	53
Atrophy	—	—	3	4	6	11	1	—	25
Cancer	—	—	1	4	10	8	1	1	25
Child-Birth	—	—	2	2	—	—	—	—	4
Consumption	2	9	34	31	44	28	5	—	153
Convulsion Fits	—	—	3	4	1	3	—	—	11
Decay (Natural) and Old Age	—	—	—	—	5	72	127	58	262
Diabetes	—	—	—	2	2	—	1	1	6
Dropsy	1	—	7	28	38	41	20	2	137
Dropsy in the Chest	—	1	3	18	34	28	16	—	100
Dysentery	—	—	1	1	2	4	4	—	12
Disease of the Stomach and Digestive Organs	—	—	5	2	8	8	1	—	26
Diseased Liver	—	2	5	24	23	21	4	—	79
Disease of the Bladder and Urinary Passages	—	—	2	4	15	23	15	—	59
Epilepsy	—	—	2	3	2	1	2	—	10
Erysipelas	—	1	—	2	3	2	2	—	10

NOSOLOGICAL TABLE—*continued.*

DISEASE.	10 to 20.	20 to 30.	30 to 40.	40 to 50.	50 to 60.	60 to 70.	70 to 80.	80, &c.	TOTAL.
Fevers, General.....	—	6	18	33	33	39	15	2	146
——— Bilious.....	—	1	4	8	9	4	1	1	28
——— Nervous	—	3	3	13	6	8	3	—	36
——— Inflammatory	—	—	—	4	6	3	2	—	15
——— Putrid.....	—	2	7	4	6	7	—	—	26
Gout.....	—	—	1	4	4	11	6	—	26
Inflammation of the Bowels..	1	2	11	13	15	25	9	1	77
——————— Lungs...	—	—	9	4	24	22	12	2	73
——————— Brain ...	—	3	7	5	5	3	—	—	23
——————— Chest, and Peripneumony	1	1	1	1	6	7	4	1	22
Palsy.....	—	1	3	8	26	42	34	2	116
Quinsy	—	—	—	1	1	1	—	—	3
Rupture of a Blood Vessel...	—	—	7	14	13	12	3	—	49
Slain in War	1	1	1	1	—	—	—	—	4
Stone.....	—	—	—	—	1	2	4	1	8
Suicide	—	1	2	3	7	2	—	—	15
Water on the Brain.....	—	—	—	1	3	1	—	—	5
	7	37	166	299	458	536	345	82	1930

TABLE

Showing the Rate of Mortality among the Members of the Equitable Society, from the Year 1768 to 1825.

Age.	Living.	Dying.	Age.	Living.	Dying.	Age.	Living.	Dying.
10	2844	11	40	2236	28	70	1028	54
11	2833	11	41	2208	28	71	974	55
12	2822	12	42	2180	28	72	919	55
13	2810	12	43	2152	29	73	864	56
14	2798	13	44	2123	30	74	808	56
15	2785	14	45	2093	30	75	752	55
16	2771	15	46	2063	30	76	697	55
17	2756	16	47	2033	31	77	642	54
18	2740	17	48	2002	32	78	588	54
19	2723	18	49	1970	33	79	534	54
20	2705	18	50	1937	35	80	480	54
21	2687	18	51	1902	37	81	426	53
22	2669	19	52	1865	39	82	373	52
23	2650	19	53	1826	41	83	321	50
24	2631	20	54	1785	41	84	271	47
25	2611	20	55	1744	42	85	224	43
26	2591	21	56	1702	43	86	181	38
27	2570	22	57	1659	44	87	143	32
28	2548	23	58	1615	45	88	111	26
29	2525	24	59	1570	46	89	85	20
30	2501	24	60	1524	46	90	65	16
31	2477	25	61	1478	46	91	49	13
32	2452	26	62	1432	47	92	36	11
33	2426	26	63	1385	48	93	25	9
34	2400	26	64	1337	49	94	16	7
35	2374	27	65	1288	50	95	9	5
36	2347	27	66	1238	51	96	4	3
37	2320	28	67	1187	52	97	1	1
38	2292	28	68	1135	53			
39	2264	28	69	1082	54			

The two preceding Tables, it must be admitted, afford some valuable information; in the last we have the law of mortality amongst persons who have been actually assured; or persons whose lives have been accepted, or were considered, at the time of their entrance upon the Society, fair lives for assurance; and from which some Tables of Annuities, &c. upon one and two lives have been published, together with that Table, by Mr. Davies, since his appointment as Actuary to the Guardian Assurance Company. Another Table of this kind that claims our attention is one given by Mr. Milne, being Table 2nd. of his scientific treatise upon these subjects, exhibiting the law of mortality at Carlisle; in the eleventh chapter of which work, that author has studied, to use his own words, "to enable the reader to judge without much trouble, what confidence the Carlisle Table is entitled to, as a just representation of the law of mortality at Carlisle; and (as far as the returns to parliament under the population acts afford the means,) throughout England."

TABLE

Showing the Probabilities of the Duration of Human Life among Males and Females, separately taken, in the Kingdom of Sweden; see Table 44th of Dr. Price's Treatise on Reversionary Payments, 1812, or Table 11th of Mr. Morgan's Treatise, 1821.

Age.	Males.		Females.		Age.	Males.		Females.		Age.	Males.		Females.	
	Living.	Dying.	Living.	Dying		Living.	Dying.	Living.	Dying.		Living.	Dying.	Living.	Dying.
0	10000	2300	10000	2090	33	4868	60	5149	65	66	2026	115	2459	120
1	7700	500	7910	518	34	4808	60	5084	65	67	1911	120	2339	120
2	7200	337	7392	350	35	4748	60	5019	60	68	1791	125	2219	120
3	6863	240	7042	250	36	4688	60	4959	56	69	1666	125	2099	120
4	6623	150	6792	135	37	4628	60	4903	56	70	1541	125	1979	150
5	6473	125	6657	120	38	4568	60	4847	56	71	1416	125	1849	140
6	6348	105	6537	105	39	4508	60	4791	58	72	1291	120	1709	150
7	6243	90	6432	85	40	4448	65	4733	65	73	1171	120	1559	160
8	6153	75	6347	70	41	4383	72	4668	75	74	1051	110	1399	150
9	6078	65	6277	60	42	4311	80	4593	76	75	941	105	1249	140
10	6013	55	6217	52	43	4231	80	4517	76	76	836	100	1109	130
11	5958	45	6165	46	44	4151	80	4441	75	77	736	90	979	120
12	5913	45	6119	40	45	4071	80	4366	72	78	646	85	859	110
13	5868	40	6079	35	46	3991	80	4294	67	79	561	80	749	100
14	5828	40	6044	35	47	3911	80	4227	65	80	481	75	649	95
15	5788	39	6009	35	48	3831	80	4162	65	81	406	70	554	90
16	5749	39	5974	40	49	3751	85	4097	70	82	336	65	464	85
17	5710	39	5934	40	50	3666	95	4027	75	83	271	60	379	80
18	5671	44	5894	42	51	3571	95	3952	80	84	211	50	299	75
19	5627	44	5852	43	52	3476	95	3872	85	85	161	40	224	55
20	5583	50	5809	43	53	3381	95	3787	85	86	121	30	169	40
21	5533	50	5766	43	54	3286	95	3702	85	87	91	22	129	30
22	5483	50	5723	43	55	3191	95	3617	85	88	69	17	99	23
23	5433	55	5680	44	56	3096	95	3532	85	89	52	14	76	18
24	5378	55	5636	45	57	3001	100	3447	90	90	38	12	58	15
25	5323	55	5591	45	58	2901	100	3357	90	91	26	9	43	12
26	5268	55	5546	50	59	2801	100	3267	100	92	17	7	31	10
27	5213	55	5496	52	60	2701	105	3167	110	93	10	6	21	8
28	5158	55	5444	55	61	2596	110	3057	118	94	4	3	13	6
29	5103	56	5389	55	62	2486	115	2939	120	95	1	1	7	4
30	5047	59	5334	60	63	2371	115	2819	120	96	0	0	3	2
31	4988	60	5274	60	64	2256	115	2699	120	97	0	0	1	1
32	4928	60	5214	65	65	2141	115	2579	120					

TABLE

Showing the Probabilities of the Duration of Human Life among Males and Females, taken collectively, in the Kingdom of Sweden; see Dr. Price's 45th Table, or Table 12th of Mr. Morgan's Treatise.

Age.	Living.	Dying.	Age.	Living.	Dying.	Age.	Living.	Dying.
0	10000	2195	33	5010	63	66	2236	118
1	7805	509	34	4947	63	67	2118	121
2	7296	344	35	4884	59	68	1997	124
3	6952	245	36	4825	58	69	1873	124
4	6707	143	37	4767	58	70	1749	127
5	6564	122	38	4709	58	71	1622	133
6	6442	105	39	4651	60	72	1489	135
7	6337	87	40	4591	65	73	1354	140
8	6250	73	41	4526	73	74	1214	130
9	6177	62	42	4453	78	75	1084	121
10	6115	54	43	4375	78	76	963	115
11	6061	45	44	4297	78	77	848	105
12	6016	42	45	4219	76	78	743	95
13	5974	38	46	4143	74	79	648	90
14	5936	37	47	4069	72	80	558	90
15	5899	37	48	3997	73	81	468	84
16	5862	40	49	3924	78	82	384	75
17	5822	40	50	3846	85	83	309	65
18	5782	42	51	3761	87	84	244	55
19	5740	43	52	3674	90	85	189	45
20	5697	47	53	3584	90	86	144	35
21	5650	47	54	3494	91	87	109	27
22	5603	48	55	3403	91	88	82	20
23	5555	48	56	3312	92	89	62	15
24	5507	50	57	3220	95	90	47	14
25	5457	50	58	3125	95	91	33	12
26	5407	52	59	3030	100	92	21	10
27	5355	54	60	2930	108	93	11	6
28	5301	55	61	2822	114	94	5	3
29	5246	55	62	2708	118	95	2	1
30	5191	59	63	2590	118	96	1	1
31	5132	60	64	2472	118			
32	5072	62	65	2354	118			

OF ANNUITIES ON LIVES,

&c. &c.

A LIFE ANNUITY continues during the existence of some assigned life or lives, and may be payable yearly, half-yearly, quarterly, momently, or at the end of any other given term less than a year; and its value will obviously depend upon the frequency of the conversion of the interest of money into principal; although in the following Tables, which involve lives not subject to a law of mortality that produces equal decrements unto the extremity of human life, the payment of the Annuity, and the conversion of interest into principal, is considered to be made only at the expiration of every year, agreeable to the observations given in the course of the present work; the payment of the Annuity may likewise be made biennially, triennially, &c.; for by taking the 2nd, 4th, 6th, 8th, &c. terms of the series de-

noting the value of an Annuity upon a given life, as given in Table I., and multiplying each term by 2, we obtain the series denoting the value of a similar Annuity upon that life, the several payments whereof must be made biennially, the interest of money being convertible into principal annually.

An Immediate Annuity is that which commences at the time the contract is entered into, and continues until the extinction of one or more of the given lives.

A Reversionary Annuity commences upon the failure of one or more of the given lives, and then continues until the extinction of one or more of the survivors. See Tables VII. VIII. IX. and X.

A Reversionary Perpetuity is not receivable until the failure of one or more of the given lives, but continues thenceforth for ever.

Annuities of the two latter kinds may be contingent; that is, the payment of the Annuity may depend upon the given lives failing according to an assigned order of survivorship. The three following cases may be given as examples of this kind, viz. an Annuity to commence at the death of C, and continue until the decease of either of two lives, A and B, should they both

survive C; an Annuity to commence at the death of B, in case C is then extinct, and continue during the life of A; an estate of a given yearly value, to be entered upon at the decease of A and B, should they fail the last of the three lives A, B and C; the claim to the reversion in these cases being uncertain, or contingent.

The most simple method of finding the value of an Annuity on any given life or on the joint continuance of any number of lives, is, to find the present value of each year's rent by reducing to numbers the several terms of which the series denoting the value of the Annuity is composed, and add the whole together; but this is a method so very laborious, that it can hardly ever with much propriety be practised, unless in order to confirm the correctness of a Table showing the value of an Annuity on single lives or joint lives and obtained by another method of computation, by finding, in this manner, the value of an Annuity on the youngest age, or ages, in the Table, and for that purpose it will readily appear that the series may be so expressed that the numerical operation may be considerably abridged.

It will therefore be evident, that if an Annuity of this kind be purchased at the prices given in Tables I. II. and III. so much of the Annuity as shall be proportional to the time intervening between the failure of the single life or joint lives and the last preceding payment will not be payable. But if it be at first agreed that such fractional part of the Annuity be recoverable, when due, to the heirs of the annuitant, it then becomes equal to an Annuity secured by land.

And if a series denoting the value of an Annuity upon a given number of joint lives by consisting of the terms denoting the present worth of one pound to be received at the end of every year unto the utmost possible extent of their joint continuance be divided into three parts; the first part containing the terms denoting the present worth of one pound to be received at the end of the 1st, 2nd, 3rd, and 4th years; the second containing the terms denoting the present worth of one pound to be received at the end of the 5th, 6th, 7th, and 8th years; and the third containing the terms denoting the present worth of one pound to be received at the end of the 9th, 10th, 11th, 12th, &c. years to the utmost extent of human life; the sum of the

four terms in the first part will give the value of an Annuity for that number of years upon the given joint lives, or the value of an immediate but imperfect, or temporary Annuity; and the second part will give the value of a remote and imperfect, or deferred and temporary Annuity upon those lives; and in like manner the third or last part will give the value of a remote or deferred Annuity upon the same lives.

The present value of any assigned year's rent or payment in this series will be the product of the probability that the given lives will jointly continue to the end of that year, and the quantity R^{-t}; R denoting one pound with its interest for a year, and t the time or number of years at the end of which the payment must be made; and will denote the value of the expectation of the given joint lives receiving one pound at the end of that time.

After this manner may the value of an Annuity upon any number of joint lives be determined; whether such Annuity be granted for the whole term of life, or for any term deferred, temporary, or deferred and temporary.

And in the same manner may the value of an Annuity upon the last survivor of any number

of lives be determined: for if an Annuity be granted upon the last survivor of those lives, then will the sum of the seven series denoting the values of Annuities upon those lives, separately and jointly, when reduced to numbers, and continued until the fractions become equal to 0, be the total value of an Annuity to continue until the three lives become extinct. And if from these series we find the total sum of the expectations upon one pound payable provided the given lives, separately and jointly, continue to the end of every year in that period during which the Annuity may be claimed by the last survivor of the three lives, we may obtain the value of an Annuity upon the last survivor of those lives for any term deferred, temporary, or deferred and temporary.

And the value of an Annuity on the joint continuance of the two last survivors of three lives, as will be hereafter shown, will be the sum of five series denoting the values of Annuities upon those lives, jointly; and from which, when reduced to numbers, we may determine the value of an Annuity upon that contingency for the whole continuance of the given lives, or for any immediate or remote part of their existence.

Other Annuities may in like manner be divided into parts of similar denominations; for by reducing to numbers the expectations upon the given lives, separately and jointly, receiving one pound at the end of every year during the time the Annuity is to continue, when extracted from the series which, when arranged with their proper signs, give the value of the Annuity for the whole duration of the given lives, we shall have the value of an Annuity upon the assigned contingency for that period only; and in this manner may the value of any reversionary Annuity, contingent or otherwise, either for the whole continuance of the given lives, or for a term deferred, temporary, or deferred and temporary, be determined.

But the value of any of these Annuities may likewise be determined by adding together the several total yearly expectations thereupon, after reducing them to numbers, instead of dividing them into series denoting the values of Annuities upon the given single and joint lives.

It will occur to the reader that after the concise method hinted at in one of the foregoing

pages for determining the numerical value of the series denoting the value of an Annuity for the whole duration of any given life or joint lives, we may determine the numerical value of the series denoting the value of an Annuity, either temporary, deferred, or deferred and temporary, upon the same single life or joint lives.

This observation is universal, and may be applied to the series composing the value of an Annuity either for the whole duration of life or for any given term, upon other conditions, on the same lives; and likewise with considerable advantage in summing up the series involved in the investigation of an Assurance of a given sum upon a given number of lives, contingent or otherwise, either for the whole duration of the given lives or for any assigned period during which the contingency of receiving the given sum may be extended, when it is proposed to determine the numerical value of the Assurance in this manner.

But the value of a remote Annuity upon three or any greater number of joint lives, being the last part, as already stated, in a series denoting the value of an Annuity for the whole

duration of life, will be the value of that part thereof that remains after the payments appertaining to the time that must expire before the Annuity can be entered upon, are taken off; and of which part the first term will be the product of the probability that such persons will live to the end of the first year of the term appointed for the continuance of the Annuity, and the quantity $R^{-(t+1)}$; R denoting one pound with its interest for a year, and t the number of years that must expire before the Annuity commences; therefore, in the case of a given number of lives, the value of such an Annuity will evidently be the value of an Annuity on the same joint lives, deferred until their ages equal the present ages and the number of years that must elapse before the Annuity commences; and will consequently be the product of the probability that the persons will live till the Annuity commences, the present value of one pound certain to be received at the end of the intermediate time, and the value of an Annuity on the joint lives of persons, as many years older than the given lives as are equal to the number of years during which the Annuity is deferred; and this will denote the value of the expectation of the given lives receiving at the end of a given term the sum which

will purchase them an Annuity for the remainder of the term unto which their joint continuance may be extended.

Having in this manner found the value of a remote Annuity upon three joint lives, being the last part of the series as above alluded to, if such value be subtracted from the value of a remote Annuity upon the three joint lives commencing at the expiration of the temporary Annuity before mentioned, and continuing to the utmost extent of human life, we shall have the value of a remote and temporary Annuity upon their joint lives; and the remote Annuity last mentioned subtracted from the value of an Annuity for the whole term of their joint continuance, will give the value of the temporary Annuity upon those lives just alluded to.

It will be evident from what has been already advanced, that the present worth of one pound to be received at the end of a given number of years, provided one at least, or two at least, of three lives be then living, will be the product of the probability that the longest, or the joint continuance of the two last survivors, of those lives, respectively, will last to the end of the given

term, and the quantity R^{-t}; this being the value of the expectation of those lives respectively receiving one pound at the end of that term.

The probability of the happening of an event subtracted from unity will give the probability of that event failing or not happening, since the sum of the probabilities of an event happening and failing amounts to unity, by which a certainty is always denoted.

The probability that a person of a given age will be alive at the end of the first year therefrom is denoted by a fraction whose denominator is the number of persons found to be living in the given Table of Observations at the age of the given life, and whose numerator is the number living at an age one year older; and the probability that he will be alive at the end of the second year is denoted by a fraction whose denominator is the number living at the age of the given life, and whose numerator is the number living at an age two years older; and the probability that he will be alive at the end of the third year is denoted by a fraction whose denominator is the number living at the age of the given life, and whose numerator is

the number living at an age three years older; and so on. The first fraction subtracted from unity will give the probability that he will not be living, but that he will be dead, at the end of the first year; and the second fraction subtracted from unity will give the probability that he will be dead at the end of the second year; and the third fraction subtracted from unity will give the probability that he will be dead at the end of the third year; and so on.

And the probability that three persons of given ages will all be alive at the end of the first year will, in like manner, be denoted by a fraction whose denominator is the product of the numbers of persons living at the ages of the given lives, and whose numerator is the product of the numbers of persons living at ages each one year older; and the probability that they will all be alive at the end of the second year will be denoted by a fraction whose denominator is the product of the numbers of persons living at the ages of the given lives, and whose numerator is the product of the numbers of persons living at ages each two years older; and the probability that they will all be alive at the end of the third year will be denoted by a fraction whose denominator is the product of the num-

bers of persons living at the ages of the given lives, and whose numerator is the product of the numbers of persons living at ages each three years older; and so on. The first fraction subtracted from unity will give the probability that they will not all be living, but that one or other of them will be dead, at the end of the first year; and the second fraction subtracted from unity will give the probability that one or other of them will be dead at the end of the second year; and the third fraction subtracted from unity will give the probability that one or other of them will be dead at the end of the third year; and so on.

And by a similar method of proceeding we may find the probability of any number of lives continuing, or not continuing, together, one, two, three, or any number of years.

If the probabilities of the happening of two, three, or any number of events that are independent of each other be multiplied together, the product will be the probability that such two, three, or any number of events will all happen; thus, if the probabilities that A and B, two of three proposed lives, A, B, and C, will be alive at the end of any given year be mul-

tiplied by the probability that C will not be then living, we shall have the probability that A and B will both be alive at the end of that year and that C will not continue to the end of it; and if the probability that A will be living at the end of any given term be multiplied by the probability that B will not be then living, and also by the probability that C will not be then living, we shall have the probability that A will be living at the end of that term and that B and C will both die previous thereto.

The total probability of the happening of an assigned event, concerning or requiring the continuance of one or more of any given lives to the end of any given term, and, as will hereafter appear to be most commonly the case, involving the extinction of one or more of them previous thereto, may be known by finding the sum of the compound probabilities, thus determined, expressing the chances of happening or ways in which such event may happen unto the extremity of, that is to say, within the limits of or during the given term.

Let the number of persons found to be living at the age of A out of a given number of per-

sons born together, and exhibited in the given Table of Observations, be denoted by a, and let a *minus* the number of them who die during one year be denoted by b, and let b *minus* the number who die during the next year be denoted by c, &c.; and let the number of persons found to be living at the age of B out of the same number of persons born together be denoted by p, and let p *minus* the number of them who die during one year be denoted by q, and let q *minus* the number who die during the next year be denoted by r, &c.; and let the number of persons found to be living at the age of C out of the same number of persons born together be denoted by A, and let A *minus* the number of them who die during one year be denoted by B, and let B *minus* the number who die during the next year be denoted by C, &c.

EXAMPLE I.

Then, if an Annuity be granted upon the longest of three lives A, B, and C, that is, if it is to continue until the extinction of the longest of the two last survivors of them, it is evident that the first payment of it will become due at the end of one year,

1st. If all the lives are in being.
2nd. If A and B are both living, and C be dead.
3rd. If B and C are both living, and A be dead.
4th. If A and C are both living, and B be dead.
5th. If A be living, and B and C be both dead.
6th. If B be living, and A and C be both dead.
7th. If C be living, and A and B be both dead.

Now if the fractions denoting the probabilities of the happening of these seven compound events be added together, we shall have the probability that one of them at least will be alive at the end of the first year; and if this be multiplied by the present worth of one pound certain to be received at the end of one year, we shall have the first year's expectation, or the present value of one pound to be received upon either all, any two, or one, of the given lives continuing alive to the end of the first year.

And if the probabilities of these events for the second year be added together, we shall have the probability that one of them at least will be alive at the end of the second year; and if this be multiplied by the present worth of one pound certain to be received at the end of the second year, we shall have the second year's expectation, or the present value of one pound

to be received if either all, any two, or one, of the given lives be living at the end of the second year.

And in a similar manner may be found the expectation of receiving one pound at the end of the third, and every following year, to the utmost extremity of human life.

Now if these yearly expectations be added together, we shall have seven perpendicular series, which being written separately with their proper signs and continued to the end of the Table, will be found to denote the values of Annuities on the given single and joint lives, the sum of which, when multiplied by the given Annuity, will be the value of the Annuity required; and of which that part that will be proportional to the time intervening between the failure of the last survivor of the three lives and the last preceding payment, will not be payable.

But the value of an Annuity upon the longest of three lives must likewise be compounded of the values of an Annuity upon the three joint lives, on the joint continuance of the two last survivors only, of the three lives, that is, after the three joint lives, and on the life of the last survivor only, of the three lives, that is, after the extinction of any two of them.

The value of an Annuity upon the longest of three lives of any common age is found also by adding together the value of an Annuity on each single life, and subtracting from that sum the value of an Annuity on the joint continuance of the two last survivors only, of the three lives, that is, after the three joint lives, and twice the value of an Annuity upon the three joint lives.

Again, if from the sum of the values of Annuities upon the single lives we subtract twice the value of an Annuity on the joint continuance of the two last survivors of the three lives, and to the remainder add the value of an Annuity on the joint continuance of the two last survivors only, of the three lives, that is, after the three joint lives, we shall obtain the value of an Annuity upon the longest of the three lives.

But the value of an Annuity upon the longest of three lives A, B and C, must evidently be the sum of the values of an Annuity on the three joint lives, on the joint lives of A and B after the death of C, on the joint lives of B and C after the death of A, on the joint lives of A and C after the death of B, on the life of A after the longest of the two lives B and C, on the life of B after the longest of the two lives A and C, and on the life of C after the longest of the two lives A

and B; this will appear also by inspecting the fractions denoting the probabilities of the happening of the aforesaid events. But this is an expression evidently reducible to one of the foregoing.

EXAMPLE II.

In the next place, let it be required to determine the value of an Annuity on the joint continuance of the two last survivors of three lives, A, B and C; that is, of an Annuity that is to continue as long as any two of them continue to exist, or until two of them become extinct.

In order to receive the Annuity at the end of the first year, one or other of four events must happen, viz.

1st. That A, B and C be then all living.

2nd. That A be dead, and that B and C be both living.

3rd. That B be dead, and that A and C be both living.

4th. That C be dead, and that A and B be both living.

Now if the fractions denoting the probabilities of the happening of these four compound events be added together, we shall have the

probability that two of them at least will be then alive; that is, the probability that either all or any two of them will be living, at the end of the first year; and if this be multiplied by the present worth of one pound certain to be received at the end of one year, we shall have the first year's expectation, or the present value of one pound to be received upon either all or any two of the given lives, continuing alive to the end of the first year.

In like manner, if the total probability for the second year be multiplied by the present worth of one pound certain to be received at the end of the second year, we shall have the present value of one pound to be received if either all, or any two, of the given lives be living at the end of the second year.

And in a similar manner may be found the expectation of receiving one pound at the end of the third, and every following year to the utmost extent of human life.

Now if these yearly expectations be added together, we shall have five perpendicular series, which, if continued to the end of the Table, will be found to denote the values of Annuities upon the given lives, jointly; viz. upon A and B, A and C, B and C, and A, B and C; the sum of which will give

the value of an Annuity of one pound to continue until the failure of the joint continuance of the two last survivors of the three lives.

EXAMPLE III.

Again, let the value be required of an Annuity on the longest of the two last survivors only, of three lives; that is, after the three joint lives.

The Annuity cannot be received at the end of the first year, unless one of the six following events should happen, viz.

1st. That A be dead, and that B and C be both living.

2nd. That B be dead, and that A and C be both living.

3rd. That C be dead, and that A and B be both living.

4th. That A and B be both dead, and that C be living.

5th. That A and C be both dead, and that B be living.

6th. That B and C be both dead, and that A be living.

Now if the fractions denoting the proba-

bilities of the happening of these six compound events be added together, we shall have the probability that one or two only of the given lives will be dead, that is, the probability that only two or one of the given lives will continue in being to the end of the first year; and if this be multiplied by the present worth of one pound certain to be received at the end of one year, we shall have the value of the first year's rent, or the expectation of receiving one pound at the end of the first year.

And in a similar manner may be found the expectation of receiving one pound at the end of the second, third, and every subsequent year to the utmost extent of human life.

Now if these yearly expectations be added together, we shall have six perpendicular series, the sum of which, if continued to the end of the Table, will be the value of the Annuity required.

Otherwise; the value of an Annuity to commence upon the extinction of three joint lives, A, B and C, and continue until the extinction of the two survivors, will evidently be the sum of the values of an Annuity on the joint lives of B and C after the death of A, on the joint lives of A and C after the death of B, on the joint lives

of A and B after the death of C, on the life of C after the longest of the two lives A and B, on the life of B after the longest of the two lives A and C, and on the life of A after the longest of the two lives B and C.

If the probability that A will be dead be multiplied into the probability of the continuance of the longest of the two lives B and C, and the probability that B will be dead be multiplied into the probability of the continuance of the longest of the two lives A and C, and the probability that C will be dead be multiplied into the probability of the continuance of the longest of the two lives A and B, we shall find that the sum of these probabilities, if the operation be continued unto the extremity of human life, will form eighteen perpendicular series; the sum of which will be equal to the sum of the values of an Annuity on the longest of the two lives A and B after the death of C, on the longest of the two lives A and C after the death of B, and on the longest of the two lives B and C after the death of A; and will be found to exceed the value of an Annuity on the longest of the two last survivors only, of the three lives, that is, after the three joint lives, by the value of an Annuity on the

life of the last survivor only, of the three lives, that is, after the extinction of any two of them.

EXAMPLE IV.

And if an Annuity be granted on the life of the last survivor only, of three lives A, B and C, that is, after the extinction of any two of them, the payment of the Annuity at the end of the first year must be made, if,

1st. A and B be both dead, and C be living.
2nd. If A and C be both dead, and B be living.
3rd. If B and C be both dead, and A be living.

Now if the fractions denoting the probabilities of the happening of these three compound events be added together, we shall have the probability that two of them will be dead and that one will be living at the end of the first year; this multiplied by the present worth of one pound certain to be received at the end of one year, will give the expectation upon one pound payable at the end of the first year.

And in a similar manner may be found the expectation of receiving one pound at the end of the second, third, and every subsequent year to the utmost extent of human life.

And if these yearly expectations be added together, we shall have twelve perpendicular series, the sum of which will be the value of the Annuity required.

But this Annuity will evidently be the sum of the values of an Annuity on the life of A after the longest of the two lives B and C, on the life of B after the longest of the two lives A and C, and on the life of C after the longest of the two lives A and B.

EXAMPLE V.

In the next place, let the value of an Annuity on the life of A after the joint lives of B and C be required.

In this case, it is evident that the Annuity will become due at the end of the first year, if,

1st. A be living, and B and C be both dead.
2nd. If A be living, B be dead, and C be living.
3rd. If A be living, C be dead, and B be living.

Now if the fractions denoting the probabilities of the happening of these three compound events be added together, we shall have

the probability that A will be living and that both or one only of the two lives B and C will be dead, at the end of the first year; this multiplied by the present worth of one pound certain to be received at the end of one year, will give the expectation upon one pound payable at the end of the first year.

And in a similar manner may be found the expectation of receiving one pound at the end of the second, third, and every subsequent year to the utmost extent of human life.

And if these yearly expectations be added together, we shall have two perpendicular series, the sum of which will be the value of the Annuity required.

EXAMPLE VI.

Again, let it be required to determine the value of an Annuity on the longest of two lives A and B after the death of C.

It is evident that, in this case, the payment of the Annuity must be made at the end of the first year if one of the three following events should take place, viz.

1st. That A and B be both living, and that C be dead.

2nd. That A be living, that B be dead, and that C be dead.

3rd. That B be living, that A be dead, and that C be dead.

Now if the fractions denoting the probabilities of the happening of these three compound events be added together, we shall have the probability that both or one only of the two lives A and B will be living and that C will be dead, at the end of the first year; this multiplied by the present worth of one pound certain to be received at the end of one year, will give the expectation upon one pound payable at the end of the first year.

And in a similar manner may be found the expectation of receiving one pound at the end of the second, third, and every subsequent year to the utmost extent of human life.

And if these yearly expectations be added together, we shall have six perpendicular series, the sum of which will be the value of the Annuity required.

It appears by inspecting the fractions denoting the probabilities of the happening of the events upon which the payment of the Annuity

the probability that A will be living and that both or one only of the two lives B and C will be dead, at the end of the first year; this multiplied by the present worth of one pound certain to be received at the end of one year, will give the expectation upon one pound payable at the end of the first year.

And in a similar manner may be found the expectation of receiving one pound at the end of the second, third, and every subsequent year to the utmost extent of human life.

And if these yearly expectations be added together, we shall have two perpendicular series, the sum of which will be the value of the Annuity required.

EXAMPLE VI.

Again, let it be required to determine the value of an Annuity on the longest of two lives A and B after the death of C.

It is evident that, in this case, the payment of the Annuity must be made at the end of the first year if one of the three following events should take place, viz.

1st. That A and B be both living, and that C be dead.

2nd. That A be living, that B be dead, and that C be dead.

3rd. That B be living, that A be dead, and that C be dead.

Now if the fractions denoting the probabilities of the happening of these three compound events be added together, we shall have the probability that both or one only of the two lives A and B will be living and that C will be dead, at the end of the first year; this multiplied by the present worth of one pound certain to be received at the end of one year, will give the expectation upon one pound payable at the end of the first year.

And in a similar manner may be found the expectation of receiving one pound at the end of the second, third, and every subsequent year to the utmost extent of human life.

And if these yearly expectations be added together, we shall have six perpendicular series, the sum of which will be the value of the Annuity required.

It appears by inspecting the fractions denoting the probabilities of the happening of the events upon which the payment of the Annuity

[illegible] Example I [illegible] that the value of that Annuity will be the sum of the values of an Annuity on the life of A after the longest of the two lives B and C, on the joint lives of A and C after the death of B, and on the joint lives of A and B after the death of C.

In like manner it appears that the value of the Annuity in Example VI. will be the sum of the values of an Annuity on the joint lives of A and B after the death of C, on the life of A after the longest of the two lives B and C, and on the life of B after the longest of the two lives A and C.

EXAMPLE VII.

With respect to a reversionary Annuity depending upon an assigned order of survivorship taking place between two assigned lives, B and C, out of the three lives, A, B and C, of any common age; if the probability that the two lives, B and C, will be dead be multiplied into the fraction denoting the probability that out of two given lives, B and C, one of them in particular, C, will fail the first, and by the probability of the continuance of the life A, we shall have the

probability of the happening of the required event; and if the operation be continued unto the utmost extent of human life, we shall find that the value of an Annuity on the life of A after the longest of two lives B and C, provided B dies last, will be the sum of four perpendicular series denoting the values of Annuities upon those lives multiplied into the fraction denoting the probability that, in the extinction of two lives of any common age, one of them in particular shall die before the other.

EXAMPLE VIII.

It may be observed, in the next place, that if the probability of the joint continuance of the three lives A, B and C, the probability that A and B will be dead and that C will be living, the probability that A and C will be dead and that B will be living, and the probability that B and C will be dead and that A will be living, be added together, and the sum be multiplied by the present worth of one pound certain to be received, we shall have the expectation requiring that all or that one only of them shall be living; and from which, if the operation be continued, will arise thirteen perpendicular series, the sum

of which will give the value of an Annuity to continue until the extinction of the last survivor of the three lives, but not to be payable whilst only two of them exist; that is, to be discontinued during the term of the joint continuance of the two last survivors of the three lives, after the three joint lives.

EXAMPLE IX.

It will be sufficient to observe, with regard to Perpetuities, or Annuities for ever after the extinction of any given lives, that if the series denoting the present worth of one pound certain to be received at the end of 1, 2, 3, 4, &c. years, *ad infinitum*, be written in a perpendicular form, and the several terms thereof be multiplied by the probability that the three lives A, B and C will all be dead at the end of the 1st, 2nd, 3rd, 4th, &c. years, respectively, we shall readily perceive that the value of an Annuity for ever after the extinction of the last survivor of three lives will be the sum of eight perpendicular series, seven of which will denote the values of Annuities upon the given single and joint lives.

EXAMPLE X.

And in a similar manner we may determine the value of an Annuity for ever after the extinction of any one of three lives; for if the probability that they will all be living at the end of any assigned year be subtracted from unity, we shall obtain the probability that they will not all be then living, but that one or other of them will be dead; and which being multiplied by the present worth of one pound certain to be then received, will give the value of the expectation thereupon for that year; and by a similar process we may find the value of the expectation upon the Annuity for every year, until at length we are carried beyond the utmost extent of human life; let us, however, continue to determine the present worth of one pound payable yearly, and we shall find that the value of an Annuity for ever after three or any number of joint lives will be the sum of two perpendicular series.

EXAMPLE XI.

Again, if the probability of the joint continuance of the two last survivors of three lives,

that is, the probability that either all or any two of them will be living, be subtracted from unity, we shall have the probability of that event not happening; and if this be multiplied by the corresponding term of the series denoting the value of an Annuity for ever, and the operation be continued, we shall quickly discover that the value of an Annuity for ever after the joint continuance of the two last survivors of three lives, that is, after the extinction of any two of them, will be the sum of six series, five of which will denote the values of Annuities upon those lives, jointly.

But the value of an Annuity for ever after the longest of three lives, of any common age, may be determined also in a manner so as to involve the values of an Annuity for ever after each single life, after each pair of joint lives, and after the three joint lives; and the value of an Annuity for ever after the joint continuance of the two last survivors of three lives, of the same common age, may, in a similar manner, be determined by means of the values of an Annuity for ever after each pair of joint lives, and twice the value of an Annuity for ever after the three joint lives.

I have just asserted that if the probability of the joint continuance of the two last survivors of three lives, that is, the probability that either all or any two of three lives will be living be subtracted from unity, we shall have the probability of that event failing or not happening; that is, the probability of the failing of the joint continuance of the two last survivors of the three lives; that is, the probability that there will not be two of them living; that is, the probability that two or all of them will be dead. Now since the probability that just two of them will be dead signifies the probability that any two will be dead and that the other will be living, it follows that the sum of the four compound fractions denoting the probabilities that A and B will be dead and that C will be living, that A and C will be dead and that B will be living, that B and C will be dead and that A will be living, and that A, B and C will all be dead, will denote or be equal to the difference between unity and the probability of the joint continuance of the two last survivors of the three lives; and whence again may be deduced the value of an Annuity for ever after the joint continuance of the two last survivors of three lives.

EXAMPLE XII.

Therefore, as a farther illustration of the method of determining the value of an Annuity for ever after the extinction of any given lives, it may be observed that, in the case of three joint lives, the Annuity may be received,

1st. If A be dead, and B and C be both living.
2nd. If B be dead, and A and C be both living.
3rd. If C be dead, and A and B be both living.
4th. If A and B be both dead, and C be living.
5th. If A and C be both dead, and B be living.
6th. If B and C be both dead, and A be living.
7th. If A, B and C be all dead.

Now if the probabilities of the happening of these events for every year of human life be added together, we shall find the sum of them to agree with the result of the method of proceeding already laid down in the case of three joint lives, by means of one compound event only; that is, by subtracting from unity the probability that the given lives will all be living.

Whence the probability that three lives will not all be living, but that one or other of them

will be dead; at the end of any given term, evidently signifies the probability that either one, any two, or all, of those lives will be dead, at the end of that term.

The probability that three lives will not all be dead, but that one or other of them will be living, at the end of any given term, on the other hand, having been already shown to signify the probability that either all, any two, or one, of them will be living, at the end of the given term,

In order to facilitate the method of determining the value of an Annuity for ever after the extinction of any given lives, it will be necessary to observe that the total value or sum (S) of the series denoting the value of an Annuity for ever may be very simply determined, in the following manner; if R be taken to denote one pound with its interest for a year, then will the value of an Annuity for ever be denoted by the series

$$\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \frac{1}{R^4} + \frac{1}{R^5} + \frac{1}{R^6} + \&c. = S;$$

whence, by multiplying the equation by R, we have $1 + \frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \frac{1}{R^4} + \frac{1}{R^5} + \&c. = SR;$ and by substituting in this equation in lieu of $\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \frac{1}{R^4} + \frac{1}{R^5} + \&c.$ its cor-

responding value, S; we have $1 + S = S\,R$; whence $S = \frac{1}{R-1}$.

Therefore the value of an Annuity for ever may be found by dividing unity by the interest of one pound for one year.

And by a similar method of proceeding we may determine the value of an Annuity for ever after a given number of years; and of an Annuity certain for a given number of years, whether that period be temporary, or deferred and temporary; and by which means we shall be enabled to determine the values of several of the reversions that will be mentioned before we enter upon the calculation of the Tables.

For if the value of an Annuity certain for six years be required; then will the sum of the first six terms of the foregoing series denote the value of the Annuity, and be thus represented;

$$\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \frac{1}{R^4} + \frac{1}{R^5} + \frac{1}{R^6} = S;$$

this equation, multiplied by R, will become

$$1 + \frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \frac{1}{R^4} + \frac{1}{R^5} = S\,R;$$

but from the first

$$\frac{1}{R} + \frac{1}{R^2} + \frac{1}{R^3} + \frac{1}{R^4} + \frac{1}{R^5} = S - \frac{1}{R^6},$$

and from the second

$$\frac{1}{R}+\frac{1}{R^2}+\frac{1}{R^3}+\frac{1}{R^4}+\frac{1}{R^5} = SR-1,$$

consequently $S-\frac{1}{R^6} = SR-1$;

whence $SR-S = 1-\frac{1}{R^6}$,

and $S = \frac{1-\frac{1}{R^6}}{R-1}$.

And the value of this temporary Annuity certain subtracted from the total value of the Annuity to continue for ever will give the value of a deferred Annuity certain to commence at the end of six years and continue for ever, or the value of a perpetuity deferred six years $= (P)^d$;

that is, $\frac{1}{R-1} - \frac{1-\frac{1}{R^6}}{R-1} = (P)^d$; whence

$(P)^d$ may be found to be equal to $\frac{\frac{1}{R^6}}{R-1}$.

And the value of an Annuity certain to commence at the end of six years and continue six years will be denoted by the series

$$\frac{1}{R^7}+\frac{1}{R^8}+\frac{1}{R^9}+\frac{1}{R^{10}}+\frac{1}{R^{11}}+\frac{1}{R^{12}} = S;$$

this equation, when multiplied by R, will become

$$\frac{1}{R^6}+\frac{1}{R^7}+\frac{1}{R^8}+\frac{1}{R^9}+\frac{1}{R^{10}}+\frac{1}{R^{11}} = SR;$$

TABLE I.

Showing the present Value of One Pound to be received at the End of any Number of Years not exceeding 100.

Years.	Value.	Years.	Value.	Years.	Value.
1	.970874	35	.355383	69	.130086
2	.942596	36	.345032	70	.126297
3	.915142	37	.334983	71	.122619
4	.888487	38	.325226	72	.119047
5	.862609	39	.315754	73	.115580
6	.837484	40	.306557	74	.112214
7	.813092	41	.297628	75	.108945
8	.789409	42	.288959	76	.105772
9	.766417	43	.280543	77	.102691
10	.744094	44	.272372	78	.099700
11	.722421	45	.264439	79	.096796
12	.701380	46	.256737	80	.093977
13	.680951	47	.249259	81	.091240
14	.661118	48	.241999	82	.088582
15	.641862	49	.234950	83	.086002
16	.623167	50	.228107	84	.083497
17	.605016	51	.221463	85	.081065
18	.587395	52	.215013	86	.078704
19	.570286	53	.208750	87	.076412
20	.553676	54	.202670	88	.074186
21	.537549	55	.196767	89	.072026
22	.521893	56	.191036	90	.069928
23	.506692	57	.185472	91	.067891
24	.491934	58	.180070	92	.065914
25	.477606	59	.174825	93	.063994
26	.463695	60	.169733	94	.062130
27	.450189	61	.164789	95	.060320
28	.437077	62	.159990	96	.058563
29	.424346	63	.155330	97	.056858
30	.411987	64	.150806	98	.055202
31	.399987	65	.146413	99	.053594
32	.388337	66	.142149	100	.052033
33	.377026	67	.138009		
34	.366045	68	.133989		

TABLE II.

Showing the Sum to which One Pound will increase when improved at Compound Interest during any Number of Years not exceeding 100.

Years.	Value.	Years.	Value.	Years.	Value.
1	1.030000	35	2.813862	69	7.687206
2	1.060900	36	2.898278	70	7.917822
3	1.092727	37	2.985227	71	8.155357
4	1.125509	38	3.074783	72	8.400017
5	1.159274	39	3.167027	73	8.652018
6	1.194052	40	3.262038	74	8.911578
7	1.229874	41	3.359899	75	9.178926
8	1.266770	42	3.460696	76	9.454293
9	1.304773	43	3.564517	77	9.737922
10	1.343916	44	3.671452	78	10.030060
11	1.384234	45	3.781596	79	10.330962
12	1.425761	46	3.895044	80	10.640891
13	1.468534	47	4.011895	81	10.960117
14	1.512590	48	4.132252	82	11.288921
15	1.557967	49	4.256219	83	11.627588
16	1.604706	50	4.383906	84	11.976416
17	1.652848	51	4.515423	85	12.335709
18	1.702433	52	4.650886	86	12.705780
19	1.753506	53	4.790412	87	13.086953
20	1.806111	54	4.934125	88	13.479562
21	1.860295	55	5.082149	89	13.883949
22	1.916103	56	5.234613	90	14.300467
23	1.973587	57	5.391651	91	14.729481
24	2.032794	58	5.553401	92	15.171366
25	2.093778	59	5.720003	93	15.626507
26	2.156591	60	5.891603	94	16.095302
27	2.221289	61	6.068351	95	16.578161
28	2.287928	62	6.250402	96	17.075506
29	2.356560	63	6.437914	97	17.587771
30	2.427262	64	6.631051	98	18.115404
31	2.500080	65	6.829983	99	18.658866
32	2.575083	66	7.034882	100	19.218632
33	2.652335	67	7.245929		
34	2.731905	68	7.463307		

TABLE III.

Showing the present Value of an Annuity of One Pound for any Number of Years not exceeding 100.

Years.	Value.	Years.	Value.	Years.	Value.
1	.970874	35	21.487220	69	28.997124
2	1.913470	36	21.832252	70	29.123421
3	2.828611	37	22.167235	71	29.246040
4	3.717098	38	22.492462	72	29.365087
5	4.579707	39	22.808215	73	29.480667
6	5.417191	40	23.114772	74	29.592881
7	6.230283	41	23.412400	75	29.701826
8	7.019692	42	23.701359	76	29.807598
9	7.786109	43	23.981902	77	29.910290
10	8.530203	44	24.254274	78	30.009990
11	9.252624	45	24.518713	79	30.106786
12	9.954004	46	24.775449	80	30.200763
13	10.634955	47	25.024708	81	30.292003
14	11.296073	48	25.266707	82	30.380586
15	11.937935	49	25.501657	83	30.466588
16	12.561102	50	25.729764	84	30.550086
17	13.166118	51	25.951227	85	30.631151
18	13.753513	52	26.166240	86	30.709855
19	14.323799	53	26.374990	87	30.786267
20	14.877475	54	26.577660	88	30.860454
21	15.415024	55	26.774428	89	30.932479
22	15.936917	56	26.965464	90	31.002407
23	16.443608	57	27.150936	91	31.070298
24	16.935542	58	27.331005	92	31.136212
25	17.413148	59	27.505831	93	31.200206
26	17.876842	60	27.675564	94	31.262336
27	18.327031	61	27.840353	95	31.322656
28	18.764108	62	28.000343	96	31.381219
29	19.188455	63	28.155673	97	31.438077
30	19.600441	64	28.306478	98	31.493279
31	20.000428	65	28.452891	99	31.546872
32	20.388766	66	28.595040	100	31.598905
33	20.765792	67	28.733049		
34	21.131837	68	28.867038	Perp.	33.333333